INSTITUTE OF DISTANCE AND OPEN LEARNING
Gauhati University
H O M E A S S I G N M E N T
M. A./M.Sc. Mathematics
(4th Semester)
2012-2013 Session

***Guidelines for Submission*:**

1. Write your name, session, roll number, the topic selected and the title of the answer *clearly on the top.*
2. Each of the two topics given in each paper will be answered as **two essays** of *not more than 500 words each*. There will be negative marking for writing in excess of the word-limit.
3. Each answer (essay) carries a weightage of **10 marks**. (10 marks x 2 essays = 20 marks).
4. Keep a margin of about 1 inch on each side of the page.
5. You can submit the essay written in your own hand-writing on clean A-4 sized paper.
6. In case you prefer to submit type-written answers, make sure that there are no typing errors which will deduct from the overall impression.
7. Do not submit commercially purchased answers as such a practice is deemed to be unfair.
8. Please submit your assignment by **15th May, 2014**.

---

401. Graph Theory (answer any two) 2×10=20

1. Give the notion of cycle space and cocycle space in a graph. Find a basis for cycle space and cocycle space. What is the rank of cycle space? 10
2. Show that a Graph G has 1 factor if and only if the order p of G is even and there is no set S of points such that the number of odd components of G-S exceeds the cardinality of S i.e. |S|. 10
3. Define adjacency matrix of a labeled Graph G. Find the eigenvalues of the adjacency matrix of K_4. What are the eigenvalues of a complete bipartite graph $K_{3,4}$? Find the spectrum of 10
   (i) K_5 (ii) $K_{m,n}$

402. Numerical Analysis (answer any two) 2×10=20

1. What is interpolation? Derive Lagrange's interpolation formula. 10
2. Prove that the divided difference of f(x) with (n+1) arguments $x_0, x_1, \ldots . x_n$ is defined by the recursion relation. 10

$$f[x_0, x_1, \ldots\ldots, x_n] = \frac{f[x_0, x_1, \ldots\ldots, x_{n-1}] - f[x_1, \ldots\ldots, x_n]}{x_0 - x_n}$$

$$= \sum_0^n \frac{f(x_i)}{(x_i - x_0)\ldots\ldots\ldots(x_i - x_{i-1})(x_i - x_{i+1})\ldots\ldots\ldots(x_i - x_n)}$$

3. Describe Bisection method for solving algebraic equation. Discuss the convergence of bisection method. What are the advantages and disadvantages of Bisection method? 10

403. (A) Functional Analysis (optional) (answer any two parts) 10+10=20

1. Vector Topologies and Metrizable Vector Topologies and Fundamental Results.
   (a) Describe Vector topology with examples and prove some Basic results and their applications.
   (b) Describe Metrizable Vector Topology and some related important theorems.
2. Weak and weak topologies and their applications.
   (a) Describe weak and weak topologies on Banach spaces with examples.
   (b) State and prove some important results, viz..
      i. If N is a normal linear space, then the closed unit sphere S^* in N^* is a compact Housdorff space in the weak topology.
      ii. Let N be a nls, and let S^* be the compact Housdorff space obtained by imposing the weak*-topology on the close unit sphere in N^*. Then the mapping $x \to F_x$, where $F_x(f)=f(x)$ for each f in S^*, is an isometric isomorphism of N into C (S^*)
   (c) Applications of these result
3. Some aspects on applications of Hahn-Banach Theorem and the Banach Alauglu Teorem on topological vector spaces.
   (a) state the theorems and show some applications.

403. (B) Fluid Dynamics (optional) 5×4=20

1. Write an essay on "Progressive waves with deduction of total energy associated with a train of surface waves".
2. Write an essay on "Group velocity and its physical significance in water waves."
3. Write an essay on "Boundary layer separation."

404. (A) Mathematical Statistics (optional) (answer any two) 2×10=20

1. What is probability distribution? Give a brief description of continuous probability distribution.
2. Describe binomial and poission distribution.
3. Give the concepts of sampling distributions what is sampling distribution of mean?

404. (B) Dynamical System and Fractal Geometry (optional) (answer any two) 2×10=20

1. Periodic Point, Bifurcation Point, and a Route from Regular to Chaotic Region
   a. Consider a model $f(x) = \lambda x(1-x), 0 < \lambda \leq 4$ and $x \in [0,1]$

      Describe a scheme of how to obtain periodic points of periods $2^0, 2^1, 2^2, \ldots\ldots$
   b. Describe a scheme of how to obtain first bifurcation point, 2nd bifurcation point, etc.
   c. Describe a technique of how to obtain accumulation point and the chaotic region.
2. Deterministic chaos and different techniques of identification of chaos.
   a. Explain deterministic chaos with examples
   b. Describe time-series analysis, lyapunov exponents and 'period-three concepts' for identification of chaos
3. Methods of numerical Analysis for solving non-linear differential equations
   a. Describe continuous dynamical systems with examples
   b. Describe with error analysis two methods for solving non-linear differential equations.

405. (A) Fuzzy Sets and Their Applications (optional) (answer any two) 2×10=20

1. Define a convex fuzzy set in R with suitable examples. Show that for a fuzzy subset A of R, the following two conditions are equivalent:
   i) $\{x \in R : A(x) \geq \alpha\}$ is a convex crisp set, $\forall \alpha \in [0,1]$
   ii) $A(\lambda x_1 + (1-\lambda)x_2) \geq \min [A(x_1), A(x_2)]$ for all $x_1, x_2 \in A$ and $\lambda \in [0,1]$.
   Can R be replaced by a real vector space X(R)? justify.
2. State the extension principle for fuzzy sets. Let $f : X \to Y$ be an arbitrary crisp function. Then for $A_i \in F(x)$ and any $B_i \in F(y)$, $i \in I$, prove that
   i) $f(\underset{i \in I}{U} A_i) = \underset{i \in I}{U} f(A_i)$
   ii) $f^{-1}(\underset{i \in I}{U} B_i) = \underset{i \in I}{U} f^{-1}(B_i)$

   Using the extension principle show that $\alpha^+[f(A)] = f(\alpha^+ A)$ where A is a fuzzy set on X. show with the help of an example that α^+ cannot be replaced by α.
3. Write notes on
   i) Fuzzy Neural Network
   ii) Fuzzy Autometa.

405 (B) General Theory of Relativity and Cosmology (optional)

1. Discuss the three crucial tests of General Relativity. 10
2. Derive the line element for Robertson-Walker non-static cosmological model. Also discuss the dynamical consequences of it. 10

* * *